Development of reduced structural theories for composite plates and shells via machine learning

Marco Petrolo, Erasmo Carrera, Matteo Filippi

MUL[2] Group, Department of Mechanical and Aerospace Engineering
Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129, Torino, Italy
{marco.petrolo, erasmo.carrera}@polito.it

Abstract

This paper presents a new approach for the development of structural models via three well-established frameworks, namely, the Carrera Unified Formulation (CUF)[1], the Axiomatic/Asymptotic Method (AAM)[2], and Artificial Neural Networks (NN)[3]. CUF and AAM provide the finite element arrays and measure the relevance of any given generalized displacement variable. The NN training makes use of the data from CUF-AAM and the outputs are the Best Theory Diagrams [4] - curves providing the minimum number of nodal degrees of freedom required to satisfy a given accuracy requirement - and the accuracy of any structural theory. The main governing equations for plate and shell finite elements via CUF are the following and lead to the implementation of any order theory,

$$\begin{aligned} &\mathbf{u}(x,y,z) = F_\tau N_i(z)\mathbf{u}_{\tau i}(x,y) => \\ &\int_{\Omega_k}\int_{A_k} (\delta\boldsymbol{\epsilon}^{k^T}\boldsymbol{\sigma}^k + \rho^k \delta\boldsymbol{u}^{k^T}\ddot{\boldsymbol{u}}^k) H^k_\alpha H^k_\beta d\Omega_k dz = 0 \quad => \quad \boldsymbol{m}^k_{\tau isj}\ddot{\boldsymbol{u}}^k_{\tau i} + \boldsymbol{k}^k_{\tau sij}\boldsymbol{u}^k_{\tau i} = 0 \end{aligned} \tag{1}$$

The inputs of the NN are combinations of the fifteen generalized displacement variables of a fourth-order model and the thickness ratio,

$$\begin{aligned} &u_x = u_{x_1} + z\,u_{x_2} + z^4\,u_{x_5} \\ &u_y = u_{y_1} + z\,u_{y_2} + z^3\,u_{y_4}, \quad h/a = 0.1, \quad => \quad [1\,1\,1\,1\,1\,1\,0\,0\,1\,0\,1\,0\,1\,0\,0\,0.1] \\ &u_z = u_{z_1} + z\,u_{z_2} + z^2\,u_{z_3} \end{aligned} \tag{2}$$

Where '1' indicates an active variable and '0' a deactivated one. The targets for the NN training are the errors over the first natural frequencies,

$$Error = \sum_{i=1}^{10} \frac{f_i/f_i^{N=4}}{10} \tag{3}$$

Where the reference frequencies are those by the full fourth-order model. The NN configuration is a multilayer feed-forward with early stopping and mean squared error as the objective function. Each layer has ten neurons. This paper adopts Levenberg-Marquardt training functions. The numerical results refer to a 0/90/0 square simply-supported spherical panel as in [4], R/a = 5. Figure 1 shows the BTD computed via two different approaches. FE refers to the full finite element approach considering 2^{15} modal analysis, i.e., one per each model stemming from the combinations of the fifteen terms of the fourth-order model. The BTD reports models that, for a given number of degrees of freedom

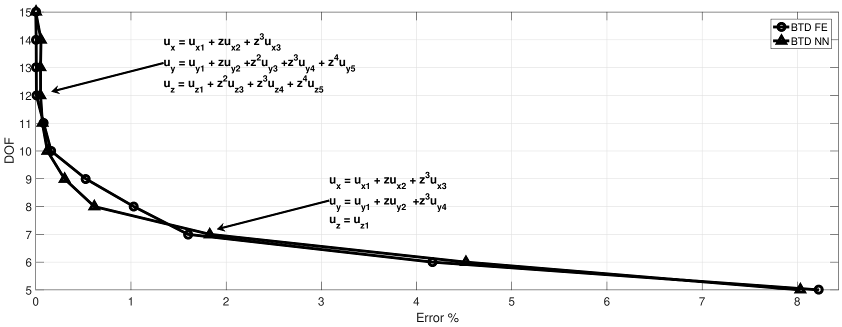

Figure 1: BTD from FE and NN a/h = 5.

(DOF), provides the minimum error. NN refers to the BTD obtained from a trained neural network. Training considered a/h = 10 and a/h = 2 and a population of some 2000 structural models. The plot reports the explicit displacement field of the 12 and 7 DOF models. Table 1 reports the error over the first ten frequencies as computed by the FE analysis and by the trained NN. PTD refers to a displacement field having full linear expansion and a parabolic term on the transverse displacement. TSDT is a model having full third-order expansions over the in-plane displacements and constant transverse one. The results suggest that

Model	DOF	FE	NN
FSDT	5	13.4	13.3
PTD	7	12.9	10.0
TSDT	9	2.6	2.0

Table 1: Mean error (%) on the first ten frequencies via FE and NN, a/h = 5.

- The use of NN is promising as a tool to evaluate the accuracy of structural theories with very high computational efficiency. Also, as the network is trained considering physical features such as the thickness, it may provide good estimates for different values of the same feature.
- Considering the structural models, the results show that, for the problem considered, models from the literature may fail in detecting the first natural frequencies with sufficient accuracy. As a general guideline, and as known from the literature, third-order in-plane variables are decisive.

References

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